Test 3 - MTH 2410

Dr. Graham-Squire, Fall 2013

Name: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Calculators are <u>not</u> allowed on any part of the in-class portion of the test. Computers and calculators are allowed on the take home part of the test (though they are not necessary), and instructions are given on that part.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 9. Total Points = 70.

1. (6 points) TRUE OR FALSE. Circle the correct answer. If false, give a counterexample or explain (briefly) why it is false. If true, no explanation is necessary (though if you are wrong, an explanation can get you some partial credit).

(a) **True or False:** If $f(x, y)$ is integrable on the rectangle $a \leq x \leq b$, $c \leq y \leq d$, then

$$\int_a^b \int_c^d f(x, y)\, dy\, dx = \int_c^d \int_a^b f(x, y)\, dx\, dy$$

(b) **True or False:**

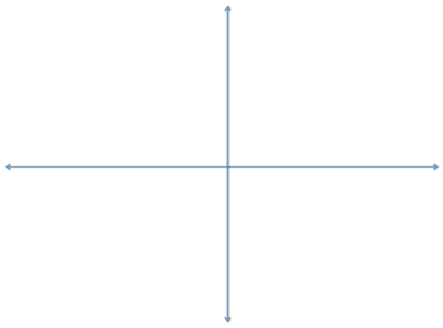
$$\int_0^4 \int_0^4 x^3 \sqrt{\sin^2(x^2y^2)}\, dy\, dx = \int_0^2 \int_0^2 x^3 \sqrt{\sin^2(x^2y^2)}\, dy\, dx + \int_2^4 \int_2^4 x^3 \sqrt{\sin^2(x^2y^2)}\, dy\, dx$$

(c) **True or False:** The volume of a sphere of radius R is given by

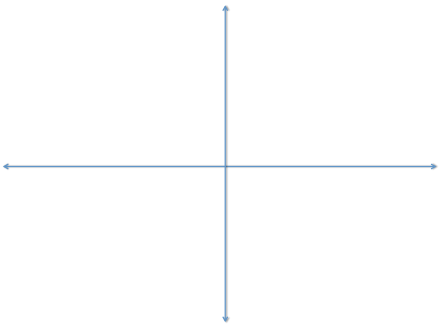
$$\int_0^{2\pi} \int_0^{2\pi} \int_0^R \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$$

For all questions involving integration, you should use whatever method and coordinate system you deem most appropriate in order to calculate the integrals. For some integrals, it may be useful to convert to polar, cylindrical, or spherical coordinates. For some integrals it may be helpful to change the order of integration, or do a change of coordinates, in order to make the integral more manageable.

2. (8 points) Evaluate the iterated integral $\int_0^1 \int_y^1 \sin(x^2)\, dxdy$.



3. (8 points) Find the volume of the region enclosed by the paraboloids $z = 16 - x^2 - y^2$ and $z = 3x^2 + 3y^2$.

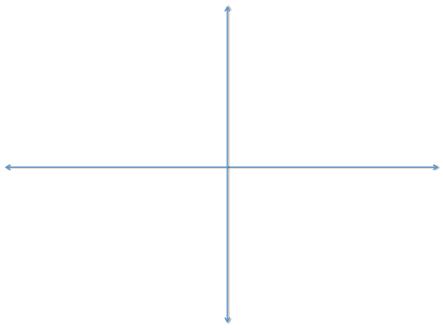


4. (8 points) Consider the triangular lamina with corners at (0,0), (1,1) and (-1,1), with density function $\rho(x, y) = y^2$.

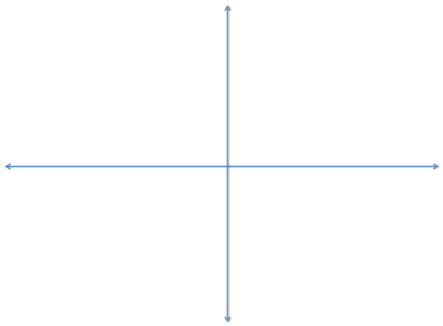
   (a) Sketch the region of integration and determine (from the shape of the region and the density functions) approximately where the center of mass should be. Explain your reasoning.

   (b) Set up, but do not integrate, the integrals that you would need in order to find the center of mass.

   (c) Calculate the center of mass to see how accurate your guess from part (a) was.

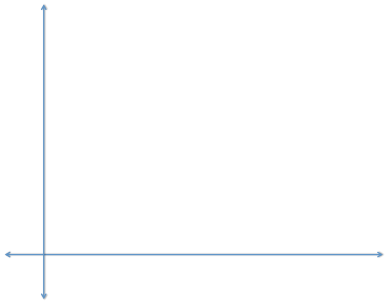


5. (8 points) Calculate the surface area of the graph of $f(x, y) = x^2 - 3xy - y^2$ over the region inside the circle $x^2 + y^2 = 4$.

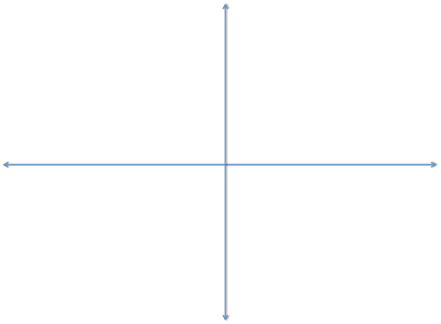


6. (8 points) Find the maximum value of the function $f(x, y) = xy$ subject to the constraint that $x^2 + 4y^2 = 16$.

7. (8 points) Calculate $\iint_R (x-y)^3 dA$, where R is the region inside the parallelogram given by connecting the points (0,0), (3,0), (5,2) and (2,2).



8. (8 points) Set up, but do not integrate, an integral to calculate the mass of the solid bounded above by $z = x^2 - y^2 + 4$, below by the xy-plane, within the region given by $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$, and with density function $\rho(x, y, z) = z$.



Extra Credit(2 points) Use $r = \sqrt{x^2 + y^2}$ and $\theta = \arctan(\frac{y}{x})$ to calculate the Jacobian for the conversion from polar coordinates to rectangular coordinates. Does your result make sense?

Take-home Name:_____

- At some point this weekend you should set aside 30 minutes to work on this question.
- You are allowed to use a calculator and a computer, including programs such as Wolfram Alpha, Sage, Maple or Grapher. All of the calculations can be done by hand (and a calculator, perhaps), and I expect you to completely show your work in order to receive full credit. The computer is only needed to confirm your results and help you check for any mistakes.
- You can use the formula sheet given in class, but you are NOT allowed to discuss this problem with anyone else, nor can you use any other resources such as notes, textbook, etc.
- You ARE allowed to look at the published documents on Sage, however.
- You can use additional paper if you need it.
- The take-home question will be due at the beginning of class on Monday.
- For this question, you should use whatever method and coordinate system you deem most appropriate in order to calculate the integral. It may be useful to convert to polar, cylindrical, or spherical coordinates. It may be helpful to change the order of integration, or do a change of coordinates, in order to make the integral more manageable.

I expect the question to take you about 20 minutes, but if it takes you longer that is okay. If it takes you 40 minutes you should stop at 40 and turn in just what you have finished up to that point. Please write down the time you start and end the question.

Start time:

End time:

9. (8 points) Find the volume of the region inside the sphere of radius 6 but lying outside the cone given by $3z^2 = x^2 + y^2$.